Precise Control Flow Reconstruction Using Boolean Logic

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ABSTRACT
This paper presents a SAT-based method for control flow graph reconstruction from executable code. The key idea of the technique is to express the semantics of each basic block in a program using Boolean logic, followed by inferring pre- and postconditions for each block through interleaved forward and backward analysis. In particular, the technique relies on register-wise value-set abstractions, which are subsequently refined using alternating forward and backward analyses. Experimental evidence shows that this approach, despite being sound, recovers the control flow graph precisely for different real-world benchmarks.

Categories and Subject Descriptors
D.2.4 [Software Engineering]: Software/Program Verification—Assertion checkers; model checking; formal methods; F.3.1 [Logics and Meanings of Programs]: Specifying and Verifying and Reasoning About Programs—Assertions; invariants; mechanical verification; F.3.2 [Logics and Meanings of Programs]: Semantics of Programming Languages—Program analysis

General Terms
Algorithms, theory, verification

Keywords
Abstract interpretation, static analysis, binary code, control flow recovery, refinement, SAT solving

1. INTRODUCTION
Ideally, verification and validation of embedded software are applied to the executable binary code of a program because the semantics of each instruction is fully specified on this level. This contrasts with high-level programming languages such as C, and thus, promises to provide more trustworthy results [2, 16, 29]. In practice, however, binary code analysis presents severe challenges to overcome so as to make an analysis feasible for practical applications, one of which is indirect control flow. Indirect control poses a so-called chicken-and-egg problem [5, 6, 22, 32]: In order to reconstruct the control flow graph from binary code, it is necessary to infer invariants that describe those registers which affect the target of an indirect jump/call. A control flow graph is, in turn, required to compute these invariants.

1.1 The Drive for Control Flow Recovery
In presence of indirect control, e.g., implemented using indirect jump instructions, the lack of a precise control flow graph often implies a drastic loss in terms of precision for any subsequent verification effort. We discuss the loss of precision incurred by spurious jump targets using an example. Consider the following macro written in C:

```c
#define SWPC(a, b) (a = b, b = a, a = b, a &= 0xf, b &= 0xf)
```

The macro swaps the contents of two different variables a and b without involving a third, based on three consecutive exclusive-or operations. Indeed, this is a well-known idiom in low-level programming [35]. In addition, the last two operations clear the upper nibbles of the results. The left-hand side of Fig. 1 shows a semantically equivalent assembly snippet after compilation for the (accumulator-based) Intel MCS-51 architecture. The compiler locates the first instruction of the macro at address 0x100 in program memory. Now assume the SWPC macro is used within a switch-case statement, say:

```c
switch (p) {
    case 10: SWPC(x, y); break;
    case 20: foo(x, y); break;
    default: bar(x, y);
}
```

For efficiency, the switch-case is compiled into a jump table, which is stored in program memory. The application looks up a comparison value and the corresponding jump target `pc′` from program memory. These two values constitute a single entry in the jump table. Address `pc′` indicates the first instruction that belongs to the respective case block. If the comparison matches, control is redirected to `pc′` using an indirect jump. In the above example, the jump table consists of the comparison values 10, 20, and #default with the corresponding jump targets: 0x100 for the implementation of SWPC and the entry addresses of functions foo() and bar(). Suppose an analysis infers a range \( \delta = [0x100, 0x105] \) for the indirect jump targets `pc′` related to SWPC by abstract interpretation [14] using intervals [11, 28]. On architectures supporting instructions of variable length (typically CISC),
edges need to be added to the CFG at the granularity of the shortest possible instruction-length. For the Intel MCS-51, this is byte-level granularity [19]. The drive for soundness forces us to add edges from the indirect jump to all concretizations of $\delta$, i.e., $0x100$, $\ldots$, $0x105$. The first value $0x100$ points to the SWPC macro. Not surprisingly, the next address $0x101$ also leads to a valid instruction. The instruction MOV A ← $0x05$, e.g., is represented by a two-byte opcode 0xE505, on the binary level. Likewise, the succeeding instruction XRL A, $0x25$ is represented by the opcode 0x5325. Yet, the second byte of MOV A ← $0x05$ and the first byte of XRL A, $0x25$ form another valid opcode, i.e., 0x0553, which represents the instruction INC $0x65$, which increments memory location 0x65. Indeed, the addresses $0x102$, $0x103$, $0x104$, and $0x105$ all indicate valid sequences of instructions (cf. Fig 1).

We call such fragments in the executable junk code as it is based on opcodes that are never executed, but only form part of the program semantics due to over-approximation. Subsequent analyses will therefore calculate invariants based on junk code as well. This, leads to a considerable loss of precision (also referred to as unbearable noise propagation [5, Sect. 1]), possibly yielding more spurious warnings. The situation is even worse if the junk code coincides with the opcodes of indirect or (un-)conditional jumps, which induces further control flow, though this is not the case in the above example. Albeit the derived interval appears to be tight at a first glance (its boundaries coincide with the jump targets), the imprecision due to the choice of intervals may lead to significant loss of analysis precision. This may render any verification efforts useless. Yet, indirect control is ubiquitous in compiler-generated as well as in handcraf7ted assembly code. Apart from switch-case statements, compilers generate indirect jumps/calls for function pointers or virtual methods. A more subtle point is given for return statements which alter the program counter according to a value that has been stored on the runtime stack. In some situations, compilers intentionally alter the runtime stack, e.g., when evaluating jump tables. Such code exhibits much similarity with code that exploits possibilities for buffer overruns. It is thus hardly possible to overestimate the value of precise invariants for such kind of program properties, thereby providing a means to distinguish harmless from harmful code.

1.2 Challenges in Control Flow Recovery

The problem of soundly analyzing indirect call and jump targets in real-world binary code is actually even more devious than discussed so far because of the presence of indirect stores. Such instructions store the contents of one register at a memory location indicated by another register. Observe that not only explicit usage of indirect store operations exhibits such behavior, but also PUSH instructions as well as direct calls, which automatically store the return address on the stack. Any indirect store operation thus needs to be handled properly. A further difficulty is the presence of bit-wise (logical) instructions in low-level code, and the frequent integration of overflowing behavior in the semantics of the program. For example, to compute a 16-bit addition on an 8-bit microcontroller architecture, compilers typically use an 8-bit addition followed by an 8-bit addition with carry. The remainder of this paper discusses an algorithm which performs alternating runs of forward and backward analysis [3] using SAT solving and integrates approaches to handle the challenges mentioned thus far. Basing the analysis on SAT solving allows to express the semantics of the analyzed programs relationally, thereby allowing the same representation to be used for both forward and backward analysis, which eases the efforts for implementing the technique.

1.3 Contributions

To summarize our work, the paper contributes a fully automatic algorithm for SAT-based control flow reconstruction. The technique itself only requires an encoding of the instruction-set semantics of the target hardware in propositional Boolean logic [9]. Based on these encodings, forward and backward value-set analyses can straightforwardly be implemented using existential quantification. This operation can, in turn, efficiently be implemented using SAT solving [10,24]. By resting the analysis on a relational semantics of the instruction set, forward and backward analyses can be executed in a uniform fashion. The algorithm itself exhibits two interesting properties: (1) it is sound in the sense that it does not miss any edges in the CFG, and (2) it turns out to be precise for many examples of typical microcontroller programs. Further, the algorithm itself is fully generic, although we demonstrate it for the Intel MCS-51 architecture.
1.4 Structure of the Paper

We present the technical steps for a single basic block in Sect. 2, and then lift the approach to program-level fixed-point iteration in Sect. 3. Afterwards, Sect. 4 presents experimental evidence, before the paper concludes with a survey of related work in Sect. 5 and a discussion in Sect. 6.

2. BLOCK ABSTRACTION

In the following, we discuss the ingredients of our technique using an example. We first show how to derive tight invariants in terms of pre- and postconditions over value sets for a single basic block in the program. Afterwards, we lift the techniques described to entire programs in Sect. 3.

To simplify presentation, we first illustrate the approach for a basic block of assembly code, which corresponds to the SWPC macro introduced in Sect. 1. In general, a basic block \( b \) is a straight sequence of instructions with a single entry point and a single exit point. In the following we denote the basic block for the SWPC macro with \( b_{\text{SWPC}} \). Our technique proceeds with representing the concrete semantics of \( b_{\text{SWPC}} \) using a propositional Boolean formula, which is composed from the instruction-level encodings of all involved instructions. This technique, which has become a standard technique in software verification due to bounded model checkers [7], is colloquially referred to as bit-blasting [8, 13, 23].

2.1 Bit-Blasting Blocks

As an example, consider the exclusive-or operations of the SWPC macro, e.g., XRL A, B, where A and B are some registers. This instruction computes the bit-wise exclusive-or of registers A and B and stores the result in A. To express the semantics of the instruction relationally, introduce bit-vectors \( a \) and \( b \) to represent the values of A and B at the entry of the block. Likewise, we introduce a bit-vector \( a' \) to represent the value of A on exit (recall that B is not altered). In what follows, let \( v[i] \) denote the \( i \)-th bit of a bit-vector \( v \).

The semantics of XRL A, B is then expressed using a Boolean formula as follows:

\[
\text{XRL A, B} := \bigwedge_{i=0}^{n-1} (a'[i] \leftrightarrow a[i] \oplus b[i])
\]

The force of such relational encodings is that they can be used to reason about program executions in both forward and backward direction (or even mixed). This means, given some values of \( a' \) on exit, it is straightforwardly possible to obtain potential values of \( a \) and \( b \) on entry. Similarly, given a value of, e.g., \( a \) on entry, it is also possible to infer suitable combinations of values of \( b \) and \( a' \). As another example, the increment \( \text{INC} \) found in the junk code in Fig. 1 can be encoded over bit-vectors \( c \) and \( c' \) as:

\[
\text{INC } c := \bigwedge_{i=0}^{n-1} (c'[i] \leftrightarrow c[i] \oplus c[j])
\]

Similar encodings can be derived for the entire instruction set of microcontrollers as discussed in [9]. In the following, we denote the encoding of an instruction \( i \) by \( \text{encode}(i) \). We can simply extend the propositional encoding of single instructions to entire basic blocks by first performing static single assignment conversion [15] (to avoid accidental coupling if a register is accessed more than once in the respective block).

Thus, for a given block \( b \), we construct a basic block formula \( \phi_b \) as a conjunction over the propositional encodings of the instructions in \( b \):

\[
\phi_b = \bigwedge_{i=\text{inst}(b)} \text{encode}(i)
\]

Here, the entry and exit of a block are denoted \( \text{fst}(b) \) and \( \text{lst}(b) \), respectively. The formula for \( b_{\text{SWPC}} \) thus consists of seven conjuncts, each of which encodes a single instruction. However, passing \( \phi_b \) to a SAT solver necessitates converting \( \phi_b \) into conjunctive normal form (CNF) first. This is done using Tseitin conversion [26, 34], which introduces fresh, existentially quantified variables \( T \) to obtain an equisatisfiable formula in CNF. We therefore denote the formula in CNF by \( \phi_b(T) \). Introducing fresh variables ensures that the size of \( \phi_b(T) \) in CNF is only a linear multiple of the size of \( \phi_b \).

2.2 Value-Set Abstraction

We apply value-set abstraction following the approach of Barrett and King [6, Sect. 3] to compute the unsigned values a register can take subject to a Boolean formula such as \( \phi_b(T) \). The key idea of their algorithm is to use alternating runs of over- and under-approximation so as to converge onto a set of values for a pre-specified register. In the following, let \( \langle v[n] \rangle = \sum_{i=0}^{n-1} 2^i \cdot v[i] \) denote the unsigned integer value of a bit-vector \( v \). Further, let \( \text{vars}(\phi_b(T)) \) denote the set of propositional variables used in \( \phi_b(T) \). To compute the values of a bit-vector \( v \in \text{vars}(\phi_b) \), however, it is necessary to eliminate all variables in \( \text{vars}(\phi_b(T)) \setminus \{v\} \) from \( \phi_b(T) \) using existential projection. To do so, we apply the SAT-based projection scheme of Brauer et al. [10], who showed how to perform existential quantifier elimination for propositional Boolean formulae using incremental SAT solving. Denote the function, which projects \( \phi_b(T) \) onto \( v \), by \( \text{proj}_v(\phi_b(T)) \).

The result of this operation is a formula in CNF. Further, observe that \( \text{vars}(\text{proj}_v(\phi_b(T))) = \{v\} \).

To illustrate, consider again the macro SWPC from the introduction. The inputs of the block are the bit-vectors \( V_{\text{in}} = \{r_{0x05}, r_{0x25}\} \) since the output of the block only depends on the values of memory locations \( 0x05 \) and \( 0x25 \) on entry. Likewise, the outputs are \( V_{\text{out}} = \{r_{0x05}, r_{0x25}, r_{0x1}\} \). Then, to determine the value set of one of the registers \( v \in V_{\text{in}} \cup V_{\text{out}} \), we compute \( \text{proj}_v(\phi_b(T)) \) and apply the algorithm of Barrett and King. The key ingredient of our algorithm is to derive preconditions on the bit-vectors in \( V_{\text{in}} \) and post-conditions on the variables in \( V_{\text{out}} \) for blocks using incremental SAT solving. After each iteration, the formula \( \phi_b(T) \) is strengthened by adding a constraint that encodes the pre- and postconditions, respectively, before eliminating existential quantifiers. Using this technique, our analysis eventually converges onto a tight approximation of the actual value sets of registers.

2.3 Deriving Pre- and Postconditions

To encode pre- and postconditions in \( \phi_b(T) \), we augment \( \phi_b(T) \) with a propositional formulae \( \psi_{\text{pre}}(V_{\text{in}}) \) or \( \psi_{\text{post}}(V_{\text{out}}) \), respectively, which encodes the constraints imposed onto \( \phi_b(T) \). Then, \( \phi_b(T) \land \psi_{\text{pre}}(V_{\text{in}}) \) describes the semantics of the block \( b \) subject to the precondition \( \psi_{\text{pre}}(V_{\text{in}}) \). For example, if an analysis infers that memory location \( 0x25 \) on entry of \( b \) — corresponding to bit-vector \( r_{0x25} \) in the formula \( \phi_b(T) \) — can take the values in \( \{0x80, 0x51\} \), we augment the formula \( \phi_b(T) \) with \( \psi_{\text{pre}}(V_{\text{in}}) \) defined as:

\[
\psi_{\text{pre}}(V_{\text{in}}) = (\langle r_{0x25} \rangle = 0x80) \lor (\langle r_{0x25} \rangle = 0x51)
\]

A CNF representation of \( \psi_{\text{pre}}(V_{\text{in}}) \) can be straightforwardly derived as before by introducing fresh variables. Based on these formal notions, we can define forward and backward interpreters to derive value-set abstractions.
Forward Block-Wise Interpreter.

We define a forward interpreter $\mathcal{F} : \text{Bool}_{\text{vars}(\phi_b(T))} \times \text{Bool}_{\text{out}} \rightarrow \text{Bool}_{\text{out}}$, where $\text{Bool}_X$ describes the class of Boolean formulae over variables $X$. The forward interpreter determines a postcondition for a formula $\phi_b(T)$ and a precondition $\psi_{\text{pre}}(V_{\text{in}})$ (which is initially true, i.e., it does not contain any constraints) as follows:

1. Let $\xi = \phi_b(T) \land \psi_{\text{pre}}(V_{\text{in}})$.
2. For each $v_{\text{out}} \in V_{\text{out}}$:
   (a) Eliminate all variables in $\text{vars}(\xi) \setminus \{v_{\text{out}}\}$ from $\xi$ using incremental SAT solving, denoted $\text{proj}_{v_{\text{out}}}(\xi)$.
   (b) Compute a value-set abstraction of $\text{proj}_{v_{\text{out}}}(\xi)$, which yields a set of values in the range $0, \ldots, 255$.
3. Store results as the postcondition $\psi_{\text{post}}(V_{\text{out}})$ of $\phi_b(T)$.

Backward Block-Wise Interpreter.

In a spirit similar to before, we define a backward interpreter $\mathcal{B} : \text{Bool}_{\text{vars}(\phi_b(T))} \times \text{Bool}_{\text{out}} \rightarrow \text{Bool}_{\text{in}}$. The backward interpreter determines a precondition for a formula $\phi_b(T)$ and a postcondition $\psi_{\text{post}}(V_{\text{out}})$ (which is initially true, too) as follows:

1. Let $\xi = \phi_b(T) \land \psi_{\text{post}}(V_{\text{out}})$.
2. For each $v_{\text{in}} \in V_{\text{in}}$:
   (a) Eliminate all variables in $\text{vars}(\xi) \setminus \{v_{\text{in}}\}$ from $\xi$ using incremental SAT solving, denoted $\text{proj}_{v_{\text{in}}}(\xi)$.
   (b) Compute a value-set abstraction of $\text{proj}_{v_{\text{in}}}(\xi)$, which yields a set of values in the range $0, \ldots, 255$.
3. Store results as the precondition $\psi_{\text{pre}}(V_{\text{in}})$ of $\phi_b(T)$.

Example.

To illustrate, assume a Boolean encoding of the precondition $\psi_{\text{pre}}^{\text{SWPC}}(V_{\text{in}})$ for $b_{\text{pre}}$, which defines the registers $\langle r_{0:0xF} \rangle \in \{10, 20, 30\} \land \langle r_{0:0x0} \rangle \in \{50, 60, 70\}$ on input. Computing $\mathcal{F}(\phi_b^{\text{SWPC}}(T), \psi_{\text{pre}}^{\text{SWPC}}(V_{\text{in}}))$ yields the postcondition:

$\psi_{\text{post}}^{\text{SWPC}}(V_{\text{out}}) = \left\{ \langle r_A' \rangle \in \{50, 60, 70\} \land \langle r_{0:0x2} \rangle \in \{4, 10, 14\} \land \langle r_{0:0x0} \rangle \in \{2, 6, 12\} \right\}$

We now apply the backward interpreter $\mathcal{B}$ to the postcondition $\psi_{\text{post}}^{\text{SWPC}}(V_{\text{out}}) = \mathcal{F}(\phi_b^{\text{SWPC}}(T), \psi_{\text{pre}}^{\text{SWPC}}(V_{\text{in}}))$ to infer a precondition for $b_{\text{pre}}$ in backward direction, which yields:

$\psi_{\text{pre}}^{\text{SWPC}}(V_{\text{in}}) = \left\{ \langle r_{0:0x0} \rangle \in \{4, 10, 14, 20, 26, \ldots, 234, 238, 244, 250, 254\} \land \langle r_{0:0x2} \rangle \in \{50, 60, 70\} \right\}$

However, note that applying $\mathcal{B}$ only leads to a coarse over-approximation of the initial precondition $\langle r_{0:0x0} \rangle \in \{10, 20, 30\}$. This is a consequence of the fact that some instructions are not invertible, e.g., the instruction ANL 0x05,#0xF in $b_{\text{pre}}$.

3. PROGRAM-LEVEL ABSTRACTION

The previous section has discussed the technical steps for computing abstractions of a single block in forward or backward direction. Most notably, these are bit-blasting, existential quantifier elimination, and value-set abstraction. This section extends the techniques described so far to perform a whole-program analysis using alternating forward and backward abstract interpretation. We discuss the algorithm for whole-program analysis using the example presented in Fig. 2. The code fragment in Fig. 2 (middle) displays a typical use of indirect control flow in embedded software. An array of function pointers is stored in a table within the program memory of the microcontroller. The functions are then indexed (after some compiler optimizations) using the parameter keyCode, which is passed to function keypress(). The results of the analysis are value sets for all registers. This includes, most notably, the two 8-bit data pointer registers DPL and DPH, which are concatenated to form a 16-bit register DP on the Intel MCS-51. Together with an additive offset stored in the accumulator A, register DP indicates the jump targets (cf. instruction at address 0x106 in Fig. 2). The value-sets of these three registers are then used to extend the control flow graph of the program.

3.1 Preprocessing

In the first step, the program file is disassembled (in a sweep linear fashion) until an JMP instruction is discovered. Then, the disassembler steps, and a control flow graph is extracted from the program fragment available thus far. A basic block representation of the program is then extracted from the control flow graph and each block is bit-blasted separately.

3.2 Worked Example

The annotated CFG in Fig. 2 shows the resulting binary code after compilation for the Intel MCS-51 microcontroller using the KEIL µVision 3 v3.23 compiler. Basic block b0x03 implements the comparison keyCode ≥ N_HNDL using a subtract instruction SUBB A, #N_HNDL. The comparison evaluates to true if the subtraction does not underflow, which is indicated by the carry flag. For the true branch, the SUBB instruction clears the carry flag and control flow is redirected to b0x0C. The constant value 16 of fail is then stored in register r0070 as the return value.

However, if the comparison evaluates to false, then SUBB underflows, sets the carry flag c, and passes control flow to b0x0F. The basic block b0x0F and its successors first calculate an offset for indexing the jump table pf. Subsequently, these blocks read the corresponding entries from the table, assign it to the data pointer DP, prepare the accumulator A and finally invoke the indirect jump JMP A+(DP) with the parameters read from program memory. In a concrete execution of the program the value of the carry flag is immediately known and determines the succeeding block.

To illustrate, suppose that b0x0F is entered with a precondition r0070 = 20. This register corresponds to the case where keyCode = 20. Then, SUBB A, #N_HNDL will determine a value of 14 for the accumulator after the subtraction, the carry flag is cleared, and b0x0C is processed next. It follows that b0x0C has a concrete input state $\langle r_0' \rangle = 14 \land \langle c \rangle = 0$, and the function keypress() returns without invoking any event handler.
Figure 2: Forward (top) and interleaved forward-backward (bottom) interpretation
3.3 Forward Interpretation

In an abstract interpretation setting, however, we perform over-approximations of possible computations of the binary code under investigation. Concrete data types are mapped to abstract domains, i.e., non-relational value sets in our setting. Suppose the abstract interpreter enters the basic block $b_{0x03}$ with the following precondition:

$$\psi^b_{\text{pre}}(V_{\text{in}}) = \{ \langle \{ r_{0x07} \} \rangle \in \{1, 2, 3, 101, 102, 103 \} \land \langle \{ c \} \rangle \in \{\text{false}\} \}$$

Applying the forward interpreter $\tilde{F}$ to compute the postcondition of $\phi_{b_{0x03}}(T)$ subject to $\psi^b_{\text{pre}}(V_{\text{in}})$, we obtain:

$$\psi^b_{\text{post}}(V_{\text{out}}) = \{ \langle \{ r'_{0x07} \} \rangle \in \{251 - 253, 95 - 97\} \land \langle \{ c' \} \rangle \in \{\text{true}, \text{false}\} \}$$

Observe that $\psi^b_{\text{post}}(V_{\text{in}})$ contains valuations of $r_{0x07}$ that either fail ($c' = \text{false}$) or pass ($c' = \text{true}$) in the comparison.

To compute a fixed point, $\psi^b_{\text{post}}(V_{\text{out}})$ is now propagated to both successors of $b_{0x03}$, i.e., $b_{0x0c}$ and $b_{0x0f}$. While the propagated postcondition $\psi^b_{\text{post}}(V_{\text{out}})$ does not drastically impact analysis precision for $b_{0x0c}$, it affects the jump targets reconstructed in the other branch. Neither of the values $\{101, 102, 103\}$ for $\langle r_{0x0a} \rangle$ can reach $b_{0x0f}$ in a concrete execution since they would cause the comparison in $b_{0x03}$ to hold (SUBB clears $c$) and control would not reach $b_{0x0f}$.

The effects of applying the forward interpreter $\tilde{F}$ on program-level are shown in the upper part of Fig. 2, yielding an approximation $\{\text{hdr2()}, \text{hdr3()}, \text{hdr4()}\}$ of the jump targets. Yet, forward analysis also yields three spurious targets, denoted ? in the figure. This imprecision stems from the block $b_{0x0f}$, which calculates the address of the corresponding entry in the jump table. The value set $\{101, 102, 103\}$ for $\langle r_{0x0a} \rangle$ on entry yields additional values $\{85, 89, 91\}$ for DPL in $b_{0x0f}$.

Reading from these addresses in $b_{0x0a}$ yields the specific byte in program memory, which depends on the remaining code of the program. This byte may thus hold an arbitrary value.

3.4 Invariant Refinement

To obtain tighter invariants, the results from forward interpretation are refined by interleaving forward and backward interpretation at conditional branching edges. Making use of the semantics of the conditional jump instruction, the outgoing edges of the respective block are labeled with the constraints from the conditional jump. To illustrate, we label the outgoing edges of $b_{0x03}$ with the constraints on the carry flag $c$ in Fig. 2. The edge $b_{0x03} \rightarrow b_{0x0f}$ is thus labeled with $c = \text{true}$, whereas the edge $b_{0x03} \rightarrow b_{0x0c}$ is labeled with $c = \text{false}$. From now on, given a set of edges $E$, we formalize edge labels by a map $\kappa : E \rightarrow \text{Bool}V_{\text{in}} \cup V_{\text{out}}$. Additionally, let $\text{pred}(b)$ and $\text{succ}(b)$ denote the set of immediate predecessors and successors of a block $b$, respectively.

The key idea of our algorithm is to use a form of depth-bounded backtracking, where the maximum depth is given by an unsigned integer $k$. To illustrate, assume that backtracking is performed starting from block $b_{0x0b}$. Then, a backtracking depth of $k = 3$ implies that the analysis backtracks along 3 predecessor blocks, i.e., it visits the blocks $b_{0x0b} \rightarrow b_{0x0d} \rightarrow b_{0x0a} \rightarrow b_{0x0f}$. The edges in the control flow graph succeeding conditional branching instructions are initialized with the constraints imposed by the respective instruction. Any other edge is labeled with the constraint $\text{true}$.

3.4.1 Algorithm

The analysis begins with a forward analysis of the program (1 and 2a). If an outgoing edge of the currently processed block contains a constraint, bounded backtracking is triggered (2b). To control the backtracking, we introduce an auxiliary variable $i$ to monitor the current backtracking depth. Backtracking then starts by propagating the refined precondition of a successor block $b_{0xcc}$ into $b$, where it is used as the postcondition so as to constrain the semantics of block $b$ (3). This step is repeated $k$ times. The refined value sets on input of the $k$th predecessor are then used as the inputs for a forward analysis (4), the outcome of which is a refined value set on input of $b_{0xcc}$ (5).

1. Apply $\tilde{F}$ to the initial block $b$ in order to derive a postcondition $\psi^b_{\text{post}}(V_{\text{out}}) = \tilde{F}(\phi_B(T), \psi^b_{\text{pre}}(V_{\text{in}}))$

2. For each $b_{0xcc} \in \text{succ}(b)$
   a. If the edge does not impose any constraints, i.e., $\kappa(b \rightarrow b_{0xcc}) = \text{true}$, then join the precondition $\psi^b_{\text{pre}}(V_{\text{in}})$ of $b_{0xcc}$ with the postcondition $\psi^b_{\text{post}}(V_{\text{out}})$ of $b$; repeat step 2 with the next successor.
   b. If $\kappa(b \rightarrow b_{0xcc}) \neq \text{true}$, continue with backtracking at step 3 and set $i = k$

3. Backtracking
   a. Rename the constraint $\kappa(b \rightarrow b_{0xcc})$ over variables in $V_{\text{in}}$ to range over variables in $V_{\text{out}}$; denote the resulting constraint by $\sigma$ and put $\xi = \phi_B(T) \land \sigma$
   b. Apply $\tilde{B}$ to $b$, derive $\eta(V_{\text{in}}) = \tilde{B}(\xi, \psi^b_{\text{post}}(V_{\text{out}}))$
   c. The precondition of $b$ is then refined by computing $\psi^b_{\text{pre}}(V_{\text{in}}) = \psi^b_{\text{pre}}(V_{\text{in}}) \cap \eta(V_{\text{in}})$, where $\cap$ denotes the intersection of value sets
   d. Decrement $i$. If $i$ is positive and $|\text{pred}(b)| = 1$, then repeat step 3 for $\text{pred}(b)$; otherwise continue with step 4

4. Forward Refinement
   a. Derive $\psi'^b_{\text{pre}}(V_{\text{out}}) = \tilde{F}(\phi_B(T), \psi^b_{\text{pre}}(V_{\text{in}}))$
   b. Increment $i$. If $i < k$ then set $b$ to $\text{succ}(b)$ and repeat step 4, otherwise continue at step 5

5. Join refined precondition
   a. Rename $\psi'^b_{\text{pre}}(V_{\text{out}})$, which ranges over output variables $V_{\text{out}}$ so that it ranges over inputs $V_{\text{in}}$; denote the formula by $\sigma'$
   b. Set $\psi^b_{\text{pre}}(V_{\text{in}}) = \sigma' \cup \psi^b_{\text{pre}}(V_{\text{in}})$
   c. Continue with next successor in step 2

3.4.2 Refinement for Branching by Example

The bottom part of Fig. 2 shows the results of the algorithm with backtracking depth $k = 1$. This section discusses how the algorithm proceeds to resolve the conditional branching instruction JC C:0x00F, which passes to control to either location 0x00C or 0x00F, depending on the value of the carry flag. To do so, it first computes a postcondition $\psi^b_{\text{pre}}(V_{\text{out}})$ for $b_{0x03}$ in forward direction in step (1). This yields the same result as before, namely:
\[
\psi_{\text{post}}^{\text{b003}}(V_{\text{out}}) = \left\{ \langle r_\alpha^b \rangle \in \{251 - 253, 95 - 97\} \land \langle r_\beta^b \rangle \in \{1 - 3, 101 - 103\} \land \langle c^e \rangle \in \{\text{true}, \text{false}\} \right\}
\]

Then succ(b003) = \{b003, b006, b007\}. Suppose the analysis proceeds with the successor b006 in step (2) and determines the edge constraint \(e = \text{true}\), thus step (2b) applies. The algorithm then sets \(i = 1\), and generates a restricted Boolean transformer \(\xi = \phi_{b006}(T) \land \sigma\) in step (3a). Renaming is applied so that the constraint \(\kappa(b006 \rightarrow b007)\) ranges over the outputs of b006 rather than the inputs of b006. Backtracking starts by applying \(F\) to \(\xi\) and \(\psi_{\text{pre}}^{\text{b003}}(V_{\text{in}})\) in step (3b), and step (3c) yields a refined precondition:

\[
\psi_{\text{pre}}^{\text{b003}}(V_{\text{in}}) = \left\{ \langle r_\alpha^b \rangle \in \{1, 2, 3\} \land \langle c^e \rangle \in \{\text{true}\} \right\}
\]

Then, \(i\) is decremented to give \(i = 0\) in step (3d). Since pred(b) = 0, the algorithm jumps to step (4) and applies the forward interpreter to the refined precondition of b003. This yields a postcondition of b003 defined as follows:

\[
\psi_{\text{post}}^{\text{b003}}(V_{\text{in}}) = \left\{ \langle r_\alpha^b \rangle \in \{251, 252, 253\} \land \langle r_\beta^b \rangle \in \{1, 2, 3\} \land \langle c^e \rangle = \{\text{true}\} \right\}
\]

Incrementing \(i\) to 1 fails the test \(i < k\) in step (4b), thus we proceed with step (5) and use the refined postcondition as precondition for the successor block by applying renaming in step (5a). Then, forward interpretation based on these new inputs gives a fresh, more precise precondition for b006, i.e.:

\[
\psi_{\text{pre}}^{b006}(V_{\text{in}}) = \left\{ \langle r_\alpha^b \rangle \in \{1, 2, 3\} \land \langle c^e \rangle \in \{\text{true}\} \right\}
\]

It is important to appreciate that, due to the refinement, the valuations in \(r_\tau\) were narrowed from \(\{1, 2, 3, 101, 102, 103\}\) to \(\{1, 2, 3\}\). This refined value set coincides with those values that reach b006 in a concrete execution of the program since the block b006 does not mutate this register. Finally, the algorithm continues with the false successor b007 = succ(b003) in step (2). Eventually, the algorithm refines the value set of \(r_\tau\) on entry of b006 to \(\{101, 102, 103\}\).

To identify the jump targets of the instruction at address 0x106 exactly, it is then necessary to propagate the refined precondition of block b006 forward. This is required to compute the value of the data pointer register DP in the block b006, where it is used twice to read a value from program memory. This is implemented using the MOVIC instructions, which read the value located at address A+DP in program memory and store it in the accumulator A. Both values are then stored in registers R1 and R2, respectively. Block b006 copies the values of R1 and R2 into the data pointer register DP and clears the accumulator. Finally, block b006 uses DP to execute an indirect jump. Forward analysis computes the precondition

\[
\psi_{\text{pre}}^{b006}(V_{\text{in}}) = \left\{ \langle DP\rangle \in \{105, 110, 115\} \land \langle DP\rangle \in \{0\} \land \langle RA\rangle \in \{0\} \right\}
\]

exactly, as desired. The three values 105, 110, and 115 indicate the addresses of the respective event handlers stored in the array of function pointers. Further, observe that the analysis thus infers that the functions hdr1, hdr5, and hdr6 are unreachable due to the precondition of the program which states that register \(r_\tau\) only contains values drawn from the set \{1, 2, 3, 101, 102, 103\}. For \(r_\tau\in\{1,2,3\}\), the functions hdr2, hdr3, and hdr4 are called, whereas keypress() returns for \(r_\tau\in\{101,102,103\}\). Finally, the program is disassembled again, this time also following the computed jump targets.

### 3.4.3 Optimizing for Indirect Reads

The previous section has demonstrated how the algorithm uses forward and backward analysis so as to resolve value sets after conditional branching instructions, which is crucial to discover the values of the data pointer DP in the indirect jump. A different problem is to resolve jump tables, though our algorithm handles this situation analogously. This section demonstrates this for the jump table generated for the switch-case statement from the introduction, shown in Fig. 3. A basic block bcmp compares the control variable, which is stored in register R7, with the comparison value \(p\) from the jump table. The instruction MOV A \(\leftarrow\) \(\Phi(A+DP)\) fetches this value from program memory and stores it in the accumulator A. This means that the jump table is indirectly addressed by the data pointer. If the comparison matches, subsequent elements in the same row of the table are addressed by some additive offset on the accumulator. However, if the comparison fails, the data pointer is incremented twice in the next block so as to point to the next comparison value. The comparison starts again for the next case branch. Otherwise, the entry address of the corresponding case branch is loaded in the pre block and the indirect jump is executed.

Forward and backward interpretation of the block bcmp thus involves reasoning about the statement MOV C A \(\leftarrow\) \(\Phi(A+DP)\), which performs an indirect read. For example, in the refinement step for the true successor we apply \(F\) to bcmp to obtain a precondition that causes the accumulator to hold the value 0 after execution of bcmp. The effects of MOV A \(\leftarrow\) \(\Phi(A+DP)\) thus need to be modeled. This could be encoded in the SAT instance by modeling the indirect read as a conditional read. Yet, this would cause the formula to explode in size. We therefore deviate from this approach and separate bcmp into three blocks so as to handle the MOVIC instruction outside the SAT solver, as a single atomic block. We thus divide bcmp into three blocks b1, b2, and b3 (cf. right-hand side of Fig. 3). Applying \(F\) to b0 simplifies the process of collecting all bytes in memory which can be accessed subject to \(\psi_{\text{pre}}^{b0}(V_{\text{in}})\). Afterwards, we apply \(F\) to b1, which yields combinations of the DP and A that yield a fixed value for A on output of the block. We then only propagate the feasible combinations of DP and A to the successor b2. We apply the same strategy for division DIV A,B and multiplication MUL A,B that are known to yield hard SAT instances.

### 4. EXPERIMENTS

We have integrated the analysis described in this paper into the [mc]square framework [31], which is written in JAVA. For SAT solving, we used SAT4J [25]. All experiments were performed on an Intel Core i5 CPU equipped with 4 GB of RAM. To evaluate the precision of our technique, we have applied it to two different sets of benchmarks for the Intel MCS-51. We have conducted the experiments with the expressed aim of answering the following two questions: (i)
The second set of benchmarks consists of programs that extensively use function pointers and pointer arithmetic.

Is the runtime tractable on non-trivial examples? (ii) How precisely are indirect jump targets recovered?

To show that our approach does not depend on the compiler used, we compiled all programs with both the Keil µVision 3 v3.23 and the Sdcc v3.0.0 compiler. All programs use indirect control flow. The first benchmark set consists of embedded C programs which implement the sample programs in the tutorial Array of Pointers to Functions in the Embedded Systems Programming magazine [20]. The programs extensively use function pointers and pointer arithmetic.

**Single Row Input** The application reads data from a bidirectional input port of the microcontroller, which is connected to several push-buttons; each button is associated to a certain handler function. The starting addresses of the handler functions are stored in an array of function pointers within the program memory.

**Keypad** The application interfaces a 3×3 keypad. Whenever a key is pressed, the column and row number is used to lookup the corresponding handler implementation in a two-dimensional function pointer array.

**Communication Link** The application handles requests transmitted over a serial link (e.g., RS-232). Valid command sequences are stored as a table in program memory. A table-lookup together with pointer arithmetics determine the index of a function pointer table that holds the callback function to handle the request.

**Task Scheduling** The application implements a low-level task scheduler. It operates on a data structure that consists of an activation interval and a function pointer. An array holds one such entry for each task in the application (5 in the implementation). Whenever a time tick is received, the application iterates over all entries in the table and checks whether the interval in the table matches the elapsed time. It then uses the stored function pointer to indirectly branch to the task.

The second set of benchmarks consists of programs that make use of non-trivial switch-case statements, another major source of indirect control flow in practice.

**Single Switch-Case** An application where the control flow of the program is controlled by a non-nested switch-case statement with 18 distinct cases and one default branch. A compact range of values to test causes the compiler to optimize the switch-case statement into a jump table. The structure of this benchmark is similar to the motivating example in Fig. 1, yet it is more complicated to analyze due to the larger jump table.

**Emergency Stop** The application implements the emergency stop function block specified by the PLCopen consortium, which has defined safety-related aspects within the IEC 61131-3 development environment to support developers of Programmable Logic Controllers. The emergency stop function [27, pp. 40–45] monitors an emergency stop button in an industrial setting.

Tab. 1 shows the experimental results for these benchmarks. The sizes of the programs range from 52 to 189 instructions overall. The table clearly shows that pure forward analysis is insufficient for recovering jump targets precisely. Most of the data pointer values from forward analysis point to locations in program memory that are out of bounds, or that do not contain any meaningful instruction. Yet, integrating backward analysis with a small bound (k=2) eliminates the redundant jump targets for all except one benchmark (Switch Case compiled using Sdcc v3.0.0). This imprecision stems from the translation applied by the compiler. Here, the jump target depends on the carry flag, which is propagated through the program. Since our value set analysis is non-relational, it fails to capture this behavior. The jump targets are thus computed for both possible values of the carry flag, thereby leading to twice the number of jump targets. It is also interesting to note that in some situations combined forward and backward analysis is cheaper than pure forward analysis. This is because the value sets propagated around the program tend to be much smaller.

From our experience, the runtimes can be further reduced by substituting SAT4J using a more competitive SAT engine such as MiniSat (the speed-up is often tenfold). Thus the given timings are very conservative; indeed SAT4J was chosen to maintain the portability of [MC]SQUARE rather than for raw performance.

5. RELATED WORK

Albeit the problem of control flow reconstruction has recently received increasing attention [5,16,18,21,22], it has been studied for more than a decade already [32,33]. The approach by De Sutter et al. [32] discusses control flow reconstruction for the Digital Alpha architecture. In this paper, they use so-called hell nodes, to which control is redirected if the jump target cannot be resolved. In contrast to our approach, their analysis starts with a conservative CFG that contains edges to hell nodes whenever indirect control is found. During the analysis, they incrementally discover additional edges to hell nodes by regular instructions. Thus, their analysis proceeds diametrically opposed to ours. In another early work, Theiling [39] includes knowledge about the compiler and the target architecture in his analysis for the TriCore and PowerPC architectures. He uses a bottom-up analysis, which is in some sense similar to our strategy. Cifuentes and Van Emmerik [12] described a technique for discovering jump tables in binary code (for decompilation) using a form of slicing and expression propagation. More recently, Holst [18] proposed to use partial evaluation of switch tables to recover indirect jump targets. This approach, however, relies on
knowledge about the compiler used so that one can identify jump tables in program memory. By way of comparison, our approach computes such information. Kinder et al. [22] have presented a formal framework that incrementally builds a control flow graph from binary code using interleaving disassembly and abstract interpretation. They show that their algorithm yields the most precise CFG that can be recovered using the abstract domain employed. Since they only use local propagation of memory values, their algorithm, which was also implemented in the tool JAKSTA [21], cannot properly resolve indirect function calls. Later, the work of Kinder et al. was extended by Flexeder et al. [16] to the interprocedural setting. Control flow reconstruction is also performed by IDAPro [17], a tool that uses linear sweep disassembly. This tool is based on several techniques such as brute-force decoding of all addresses, pattern matching, and so on. However, the control flow graph provided by IDAPro is unsafe as shown in [4,21]. Just recently, Bardin et al. [5] have tackled the problem using bounded value sets (k-set propagation). They have observed that typically only few constraints need to be tracked to resolve jump targets precisely. Based on this observation, they have thus developed an algorithm that takes care of what they call precision requirements. In this approach, refinement is used to control the k-bounds, based on backward propagation of precision requirements. A similar degree of locality in the analysis can also be seen in our framework, with the key difference that we adjust the backward propagation depth.

Table 1: Experimental results for pure forward analysis as well as combined forward and backward analysis

<table>
<thead>
<tr>
<th>Binary Program</th>
<th>Compiler</th>
<th>locC</th>
<th>instrB</th>
<th>JT</th>
<th>(\mathcal{F}) interpreter</th>
<th>(\mathcal{F} + \mathcal{B}) interpreter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>RT</td>
<td>FT</td>
</tr>
<tr>
<td>Single row input</td>
<td>KEIL</td>
<td>80</td>
<td>67</td>
<td>5</td>
<td>2401</td>
<td>2395</td>
</tr>
<tr>
<td></td>
<td>SDCC</td>
<td>80</td>
<td>67</td>
<td>5</td>
<td>460</td>
<td>454</td>
</tr>
<tr>
<td>Keypad</td>
<td>KEIL</td>
<td>113</td>
<td>113</td>
<td>9</td>
<td>3844</td>
<td>3835</td>
</tr>
<tr>
<td></td>
<td>SDCC</td>
<td>113</td>
<td>113</td>
<td>9</td>
<td>1508</td>
<td>1499</td>
</tr>
<tr>
<td>Communication Link</td>
<td>KEIL</td>
<td>111</td>
<td>164</td>
<td>8</td>
<td>6889</td>
<td>6881</td>
</tr>
<tr>
<td></td>
<td>SDCC</td>
<td>111</td>
<td>164</td>
<td>8</td>
<td>84</td>
<td>76</td>
</tr>
<tr>
<td>Task Scheduler</td>
<td>KEIL</td>
<td>81</td>
<td>105</td>
<td>5</td>
<td>&gt;1000</td>
<td>&gt;995</td>
</tr>
<tr>
<td></td>
<td>SDCC</td>
<td>81</td>
<td>105</td>
<td>5</td>
<td>97</td>
<td>97</td>
</tr>
<tr>
<td>Switch Case</td>
<td>KEIL</td>
<td>82</td>
<td>166</td>
<td>19</td>
<td>&gt;5000</td>
<td>&gt;4981</td>
</tr>
<tr>
<td></td>
<td>SDCC</td>
<td>82</td>
<td>166</td>
<td>19</td>
<td>3304</td>
<td>3285</td>
</tr>
<tr>
<td>Emergency Stop</td>
<td>KEIL</td>
<td>138</td>
<td>150</td>
<td>9</td>
<td>768</td>
<td>759</td>
</tr>
<tr>
<td></td>
<td>SDCC</td>
<td>138</td>
<td>150</td>
<td>9</td>
<td>256</td>
<td>247</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>locC</th>
<th>Lines of C code</th>
</tr>
</thead>
<tbody>
<tr>
<td>instrB</td>
<td>Number of assembly instructions</td>
</tr>
<tr>
<td>JT</td>
<td>Number of jump targets</td>
</tr>
</tbody>
</table>

RT ... Number of recovered targets
FT ... Number of recovered false targets
RS ... Number of refinement steps applied
k ... Backtracking depth
Time ... Analysis time in seconds

6. CONCLUDING DISCUSSION

This paper argues that Boolean logic and SAT solving provide a promising means to reason about rather intricate examples of binary code. In particular, it shows that alternating executions of forward and backward analysis are useful to soundly recover jump targets in the value-set abstract domain. Expressing the concrete semantics of a program in the computational domain of propositional Boolean formulae allows to define its semantics relationally. This dovetails with our approach since the same encodings can thus be used for forward as well as for backward analysis, thereby sidestepping the difficulty of designing backward transformers [30]. Moreover, the approach benefits from the progress on state-of-the-art SAT solvers, which can easily decide satisfiability of structured problems involving thousands of variables and clauses. Indeed, the problems we confront the solver with can almost be seen as trivial by nowadays standards. We have explained and presented the techniques in the context of jump target recovery. Though this is a compelling problem, it is important to appreciate that our methods are merely independent of it. They can, similar to the handling of indirect reads in Sect. 3.4.3, also be used to model indirect stores.
7. ACKNOWLEDGMENTS

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8. REFERENCES